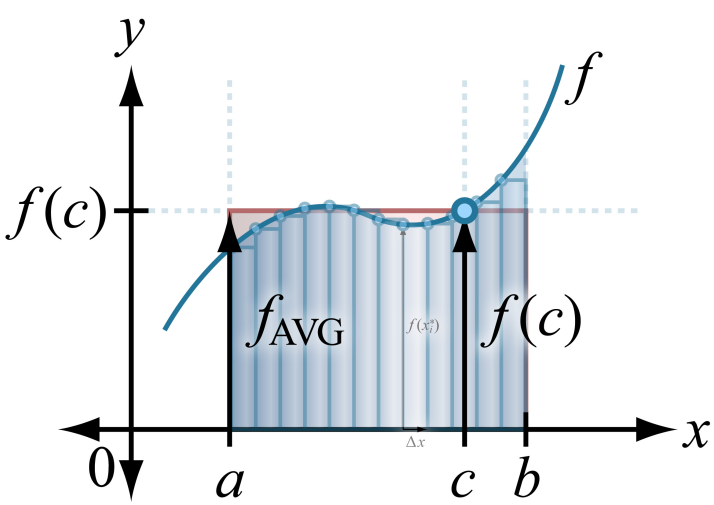
Using integrals to compute planar areas

Average values of functions



$$\int_a^b f(x)\,\mathrm{d}x \approx \sum_{i=1}^{n} f(x_i^*)\Delta x$$

$$= \left[\sum_{i=1}^{n} f(x_i^*)\right]\Delta x$$

$$= \frac{1}{n}\left[\sum_{i=1}^{n} f(x_i^*)\right] n\Delta x$$

$$\int_a^b f(x)\,\mathrm{d}x = \underbrace{\frac{1}{n}\left[\sum_{i=1}^{n} f(x_i^*)\right]}_{\text{avg. of}\{f(x_i^*)\}}(b-a)$$

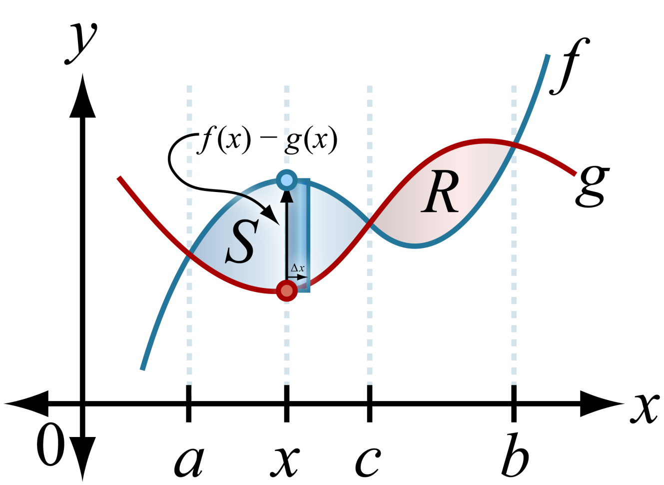
Definition
The average value of a function f on an interval $[a, b]$ is

$$f_{\mathrm{AVG}} := \frac{1}{b-a}\int_a^b f(x)\,\mathrm{d}x$$

Mean value theorem for integrals
If function f is continuous on $[a, b]$, then there is at least one value of $x = c$ on $[a, b]$ where

$$f(c) = f_{\mathrm{AVG}}$$

Areas bounded between plots of functions



1. Draw a large figure. Large means that there is enough space to mark the figure up with comments.
2. Draw a representative rectangular strip.
3. To use horizontal strips instead of vertical strips, replace x with y, TOP with RIGHT, and BOTTOM with LEFT throughout the following formulas.

$$\Delta A(x) = [f_{\mathrm{TOP}}(x) - f_{\mathrm{BOTTOM}}(x)]\Delta x$$

$$A = \int_a^b [f_{\mathrm{TOP}}(x) - f_{\mathrm{BOTTOM}}(x)]\,\mathrm{d}x$$